Evaluate $\int \sin^5 x \cos^4 x \, dx$.

$u = \cos x$ 1

$du = -\sin x \, dx$

SCORE: _____ / 5 PTS

$$\int -\sin^4 x \cos^4 x \cdot -\sin x \, dx$$

$$= -\int (1-u^2)^2 u^4 \, du \quad 1\tfrac{1}{2}$$

$$= \int (-u^4 + 2u^6 - u^8) \, du$$

$$= -\tfrac{1}{5}u^5 + \tfrac{2}{7}u^7 - \tfrac{1}{9}u^9 + C \quad 1\tfrac{1}{2} \quad = -\tfrac{1}{5}\cos^5 x + \tfrac{2}{7}\cos^7 x - \tfrac{1}{9}\cos^9 x + C \quad 1$$

Evaluate $\int x \sec^2 x \, dx$. $= \underbrace{x \tan x}_{2} + \underbrace{\ln|\cos x|}_{3} + C$

SCORE: _____ / 5 PTS

u	dv
x	$\sec^2 x$
1	$\tan x$
0	$-\ln\|\cos x\|$

Evaluate $\int e^{-3x} \sin 4x \, dx$. $= -\frac{1}{3} e^{-3x} \sin 4x - \frac{4}{9} e^{-3x} \cos 4x$

SCORE: _____ / 5 PTS

u	dv
$\sin 4x$	e^{-3x}
$4\cos 4x$	$-\frac{1}{3}e^{-3x}$
$16\sin 4x$	$\frac{1}{9}e^{-3x}$

$$- \frac{16}{9} \int e^{-3x} \sin 4x \, dx$$

1 EACH

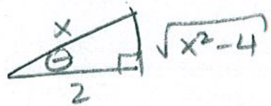
$$\frac{25}{9} \int e^{-3x} \sin 4x \, dx = -\frac{1}{3} e^{-3x} \sin 4x - \frac{4}{9} e^{-3x} \cos 4x$$

$$\int e^{-3x} \sin 4x \, dx = \frac{-3}{25} e^{-3x} \sin 4x - \frac{4}{25} e^{-3x} \cos 4x + C$$

Evaluate $\int x^2 \sqrt{x^2 - 4}\, dx$.

SCORE: _____ / 8 PTS

$x = 2\sec\theta \longrightarrow \sec\theta = \frac{x}{2}$



$dx = 2\sec\theta\tan\theta\, d\theta$

$\int (2\sec\theta)^2 \sqrt{4\sec^2\theta - 4}\,(2\sec\theta\tan\theta)\, d\theta$

$= 16\int \sec^3\theta\tan^2\theta\, d\theta$

1 EACH EXCEPT AS NOTED

$= 16\int \sec^3\theta(\sec^2\theta - 1)\, d\theta$

$= 16\int \sec^5\theta\, d\theta - 16\int \sec^3\theta\, d\theta$

$= 16\left(\frac{1}{4}\sec^3\theta\tan\theta + \frac{3}{4}\int \sec^3\theta\, d\theta\right) - 16\int \sec^3\theta\, d\theta$

$= 4\sec^3\theta\tan\theta - 4\int \sec^3\theta\, d\theta$

$= 4\sec^3\theta\tan\theta - 4\left(\frac{1}{2}(\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|)\right) + C$

$\frac{1}{2}$ $= 4\sec^3\theta\tan\theta - 2\sec\theta\tan\theta - 2\ln|\sec\theta + \tan\theta| + C$

$= 4\left(\frac{x}{2}\right)^3 \frac{\sqrt{x^2-4}}{2} - 2\left(\frac{x}{2}\right)\frac{\sqrt{x^2-4}}{2} - 2\ln\left|\frac{x}{2} + \frac{\sqrt{x^2-4}}{2}\right| + C$

$= \frac{1}{4}x^3\sqrt{x^2-4} - \frac{1}{2}x\sqrt{x^2-4} - 2\ln\left|x + \sqrt{x^2-4}\right| + C$ $\frac{1}{2}$

Prove the reduction formula $\int \sin^n u\, du = -\frac{1}{n}\sin^{n-2} u \cos u + \frac{n-1}{n}\int \sin^{n-2} u\, du$. SCORE: _____ / 7 PTS

NOTE: You must show how to get this formula.
You will receive 0 credit if your "proof" is differentiating both sides of the equation.

U	dv
$\sin^{n-1} u$	$\sin u$
$(n-1)\sin^{n-2} u \cos u$	$-\cos u$

1½ EACH EXCEPT AS NOTED

$$\int \sin^n u\, du = -\sin^{n-1} u \cos u + (n-1)\int \sin^{n-2} u \cos^2 u\, du$$

$$= -\sin^{n-1} u \cos u + (n-1)\int \sin^{n-2} u (1-\sin^2 u)\, du$$

$$= -\sin^{n-1} u \cos u + (n-1)\int \sin^{n-2} u\, du - (n-1)\int \sin^n u\, du$$

$$n\int \sin^n u\, du = -\sin^{n-1} u \cos u + (n-1)\int \sin^{n-2} u\, du$$

$$\int \sin^n u\, du = -\frac{1}{n}\sin^{n-1} u \cos u + \frac{n-1}{n}\int \sin^{n-2} u\, du$$ 1